



# Nonlinear dynamics of the 3D Alfvén waves in plasma of ionosphere and magnetosphere<sup>☆</sup>

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## ABSTRACT

The nonlinear dynamics of the 3D solitary Alfvén waves propagating nearly parallel to the external magnetic field in plasma of ionosphere and magnetosphere, which are described by the model of the 3-DNLS equation, is studied analytically and numerically. Under the assumption of negligible dissipative effects the analytical estimates and the sufficient conditions for the stability of 3D solutions of the 3-DNLS equation are obtained, based on the transformational properties of the system's Hamiltonian for the whole range of the equation coefficients. On the basis of asymptotic analysis the solutions asymptotics are presented. To study the evolution of the 3D Alfvén solitary waves including propagation of the Alfvén waves' beams in a magnetized plasma the equation are integrated numerically using the simulation codes specially developed. The results show that the 3-DNLS equation in non-dissipative case can have the stable 3D solutions in form of the 3D Alfvén solitons, and also on a level with them the 3D solutions collapsing or dispersing with time. In terms of the self-focusing phenomenon the results obtained can be interpreted as the formation of the stationary Alfvén wave beam propagating nearly parallel to magnetic field, or Alfvén wave beam spreading, or the self-focusing of the Alfvén wave beam. The influence of the dissipation in the medium on structure and character of evolution of 3D Alfvén waves is studied.

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## 1. Introduction. Basic equations

We study stability and dynamics of the multidimensional soliton-like Alfvén structures forming on the low-frequency branch of oscillations in the ionospheric and magnetospheric plasma which are described by equation (Belashov, 2011)

$$\partial_t u + A(t, u) u = f, \quad f = \kappa \int_{-\infty}^x \Delta_{\perp} u dx, \quad \Delta_{\perp} = \partial_y^2 + \partial_z^2 \quad (1)$$

when for

$$A(t, u) = 3s |p|^2 u^2 \partial_x - \partial_x^2 (i\lambda + \nu) \quad (2)$$

it falls into 3D derivative nonlinear Schrödinger (3-DNLS) equation class. Derivation of Eq. (1) with differential operator (2) was presented in detail in (Belashov and Vladimirov, 2005) with use of the same approach and conditions as in (Petviashvili and

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Pokhotelov, 1992; Pokhotelov et al. 1996a; 1996b). In the case when  $\beta \equiv 4\pi nT/B^2 > 1$  the 3-DNLS Eqs. (1), (2) describes dynamics of the finite-amplitude Alfvén waves propagating nearly parallel to homogeneous magnetic field  $\mathbf{B}$  for  $u = h = (B_y + iB_z)/2B_0(1 - \beta)$ ,  $h = B_{\perp}/B_0$  where  $p = (1 + ie)$ , and  $e$  is the “eccentricity” of the polarization ellipse of the Alfvén wave (Belashov, 1997),  $\nu = \frac{\rho_0}{2\rho} (c_{\infty}^2 - c_0^2) \tau \int_0^{\infty} \xi \varphi(\xi) d\xi$  defines the logarithmic damping rate, and it is the characteristic rate of the relaxation damping of the “sound” wave (Belashov and Vladimirov, 2005). Here  $\rho$  is perturbed plasma density  $\left( \lim_{|x| \rightarrow \infty} \rho = \rho_0 \right)$ ,  $c_{\infty}$  and  $c_0$  are the velocities of the high and low-frequency “sound” mode (the last one coincides with  $c_0 = (T/m_i)^{1/2}$ ) and  $\varphi(t, \tau)$  is the function defining the relaxation process. The upper and lower signs of  $\lambda = \pm 1$  correspond to the right and left circularly polarized wave, respectively; the sign of nonlinearity is accounted by the factor  $s = \text{sgn} (1 - p) = \pm 1$  in the nonlinear term; and  $\kappa = -r_A/2$ ,  $r_A = v_A/\omega_{oi}$ .

Eqs. (1), (2) are not completely integrable. Therefore, to study the stability of multidimensional solitons we use the method developed in (Belashov, 1999) and investigated the Hamiltonian bounding with its deformation conserving momentum by solving